Multihoming in Ridesharing Markets: Welfare and Investment

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Abstract

We propose a model of competing ridesharing platforms that allows us to analyze the impact of multi-homing on drivers and riders. We show that when platforms are symmetric, multi-homing is socially superior to single-homing, providing higher surplus to both drivers and riders. However, when platforms are asymmetric, new harms arise: multi-homing decreases the incentives for a platform to invest in more efficient matching technology, which may ultimately reduce welfare for riders and drivers in the long term. Furthermore, multi-homing increases the risk of an efficient platform monopolizing the market, which would hurt both riders and drivers. Thus, multi-homing may offer short-term benefits but long-term harms to all market participants.

Keywords: ridesharing, multihoming, competition, investment, efficiency

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1 Introduction

The rise of ridesharing platforms has fundamentally changed consumer transportation. According to Statista (2019), 1.7 billion trips were taken on Uber in the first quarter of 2020, and 103 million users around the world use Uber every month. Uber is neither alone nor dominant in every market. Lyft, Ola, Grab, Didi, Bolt, Gojek and others compete with Uber. Collectively, ridesharing platforms constitute a $41 billion industry.

A feature of ridesharing (and gig economies), absent in conventional labor markets, is *multi-homing*. Drivers can drive for multiple platforms (e.g. Uber and Lyft) at the same time. Does multi-homing in ridesharing markets benefit market participants? If so, which participants—and under what circumstances? These questions are particularly salient when evaluating the consequences of classifying drivers as employees, which would prevent them from driving for multiple ridesharing platforms. In this way, the classification of drivers as employees would eliminate multi-homing.

Multi-homing has been studied in other platform markets, such as credit card markets and online marketplaces. Ridesharing differs from these platforms because of the importance of congestion effects. An important aspect of this paper is analyzing multi-homing's effect on *latency*, the delay cost caused by supply-demand imbalances. For example, a rider may have to wait for a driver because there are too many riders on the platform. Latency is a non-price toll on drivers and riders. As a result, supply-demand imbalances can be resolved either by prices or by delay.

We propose a model of ridesharing, in which drivers and riders are affected both by the price of a ride, as well as the platform's latency. Intuitively, lower prices will lead to longer delays by reducing driver supply and increasing rider demand—a similar effect holds for higher prices. Therefore, in many cases price and delay costs have opposing effects on welfare. To resolve these opposing effects we analyze two scenarios: one where drivers must commit to driving for only one platform (single-homing), and one where drivers can drive for both platforms (multi-homing).

We find that both drivers and riders are better off under multi-homing compared to single-homing. This improvement comes from the opposing direction of price effects and latency effects. Riders are affected both by lower prices and by higher rider latency, but they benefit more from lower prices than they suffer from latency, resulting in a net benefit to riders from multi-homing. Similarly, drivers are affected both by lower wages and by lower driver latency, but they benefit more from lower latency than they suffer from lower wages, resulting in a net benefit to drivers from multi-homing. The intuition for this result is that for riders, wait-time is an inconvenience separate from the price they pay for the ride. However, for drivers, wait-time prevents them from taking on future rides, and thus decreases their overall wage. Since latency has a multiplicative effect on driver utility but only an additive effect on rider utility, multi-homing enhances welfare for both sides by giving drivers latency reductions and giving passengers price reductions (which they value more than drivers do).

While the multi-homing equilibrium in our model is superior to the single-homing equilibrium for both drivers and riders, it is not unambiguously better in an enlarged model where firms have asymmetric costs of matching. In this case, we find that under multi-homing, a firm's profit *declines* when it becomes more efficient than its competitor. This is caused by the externalities generated by multi-homing drivers: becoming more efficient at catching trips through firm A's superior matching technology, frees them to drive for firm B, which increases the demand on firm B's platform as well and ultimately reduces firm A's profits. Thus, we demonstrate a novel harm of multi-homing: it may reduce a firm's incentives to invest in better matching technology, a disincentive that could prevent welfare improvements for riders and drivers in the long-term.

Furthermore, asymmetry introduces a new harm of multi-homing: the risk of monopolization. Under an asymmetric model, it is possible for an efficient firm to drive its inefficient competitor to exit the market. We find that under multi-homing, the level of efficiency required to monopolize the market is substantially lower than it is under single-homing: in other words, multi-homing makes competition more unstable and reduces the barrier to one firm monopolizing the market with its efficiency advantage. Furthermore, we find that this monopolization is welfare-reducing: the monopoly induced under multi-homing is worse for both drivers and riders than multi-homing competition, while the monopoly induced under single-homing is actually better for both drivers and riders than multi-homing competition. The intuition is that multi-homing makes competition more intense and reduces firm profits: thus, firms are less protected from being driven out of the market. Under single-homing, the competition is less intense—in order to drive its competitor out of the market, a firm has to be so efficient that its monopoly is actually better for riders and drivers than the competitive benchmark.

The rest of this paper is organized as follows. Section 2 places this paper's results in context of prior work. Section 3 introduces the duopoly model of ridesharing competition. Section 4 contrasts single-homing and multi-homing, proving that multi-homing is better for riders and drivers under our model. Section 5 extends the model to consider the possibility of firms being asymmetrically efficient, and proves the results on investment and monopolization. Section 6 concludes with remarks about the scope for future work. Proofs of all results can be found in the appendices.

2 Prior Work

Ridesharing markets are a particular case of two-sided platform markets, which have been extensively studied. Seminal papers include Rochet and Tirole (2003), Evans and Schmalensee (2005) Armstrong (2006), and Armstrong and Wright (2007). Some of this work includes some analysis of multi-homing in two-sided platforms (Gabszewicz and Wauthy 2004; Choi 2010): however, the specific focus on latency that is essential to ridesharing is absent. The literature on ridesharing can be divided into three categories. The last of these categories is most relevant to this paper.

In the first category are papers that quantify the benefits to drivers of participation in ridesharing platforms. It documents that the ridesharing labor market sees substantially larger utilization (proportion of driver time spent transporting consumers) than the traditional taxi industry (Cramer and Krueger 2016). Furthermore, ridesharing, unlike taxi services, drivers pay the platform a commission per ride, rather than a fixed fee (lease). Angrist, Caldwell, and Hall (2017) find substantial willingness-to-pay for this commission model over a fixed lease, indicating that ridesharing benefits drivers. (Hall and Krueger 2018), finds that Uber's driver supply is highly elastic, with driver hourly earnings remaining effectively constant even with price changes due to compensating supply changes.

The second category focuses on the short run dynamic pricing aspects of ridesharing that is most prominent in the popular imagination. Competition is ignored so that attention can be paid to the role of prices in managing the supply of drivers and the demand of passengers. Examples are Castillo, Knoepfle, and Weyl (2017), Banerjee, Johari, and Riquelme (2016), Cohen et al. (2016) and Chen and Sheldon (2016).

The third and last category consists of papers devoted to understanding how competition in ridesharing shapes outcomes. Our finding that multi-homing is welfare-enhancing for riders and drivers contrasts with Bernstein, DeCroix, and Keskin (2018) and Liu, Loginova, and Wang (2017). The first finds that while individual drivers have an incentive to multi-home, all drivers are worse off under multi-homing. Similarly, Liu, Loginova, and Wang (2017) find that full multi-homing on either side of the market leads to the same outcomes as a monopoly market, suggesting a welfare decline. This difference with our conclusions arises because wages and prices are endogenous in our model. Bernstein, DeCroix, and Keskin (2018), focused on surge pricing, take driver wages to be fixed, and propose a model wherein firms maximize volume rather than profit. Liu, Loginova, and Wang (2017) hold prices, supply and demand to be fixed and emphasize the congestion variation between multi-homing and single-homing. Thus, these papers are more relevant in the short-run while our model is more applicable to studying the long-run equilibrium outcome of a ridesharing market under both single-homing and multi-homing structures. Hence, multi-homing may be welfare-reducing in the short-term, but welfare-enhancing in the long-term when supply, demand, prices and wages can adjust in equilibrium.

In our model of ridesharing markets, we focus on cost and latency for both drivers and riders. This contrasts with Bryan and Gans (2019), who model ridesharing competition with a Hotelling type model that emphasizes the geographic causes of latency. Their analysis differs in two ways: first, by virtue of analyzing a Hotelling model, they assume volume to be fixed whereas our model allows volume to be elastic with regards to price. Empirical analysis suggests Uber's demand and supply are both highly elastic with respect to price (Uber 2014; Chen and Sheldon 2016), which makes this elasticity important to study. Second, their emphasis on geographical distribution of riders and drivers can be seen as complementary to our emphasis on supply-demand imbalances, insofar as geographic distributions can lead to supply-demand imbalances, but supply-demand imbalances may be generated by other factors (population, efficiency of matching, pricing, etc).

Also related to our paper is Nikzad (2017), who analyzes the effect of thickness in service platform labor markets. Our analysis complements Nikzad's in two ways: first, we consider two alternative scenarios where drivers can multi-home and where they cannot, whereas Nikzad considers only the case where drivers can multi-home. Second, Nikzad focuses on varying the thickness of the labor market, whereas we allow thickness to be determined at equilibrium through market-clearing conditions. Thus, we focus more on the effect of multi-homing rather than the effect of market thickness, and our analysis is more descriptive of long-run equilibrium.

3 Model

Let d_i denote rider demand on platform i, s_i denote the driver supply on platform i, w_i denote driver wages on platform i, and let p_i be the price paid by riders of platform i. We introduce a *latency term* l_i, determined endogenously, representing the waiting cost for riders induced in this matching market. Likewise, driver latency—the waiting cost for drivers—is defined as the inverse, $l_i^d = \frac{1}{l_i}$. This is because intuitively, driver waiting costs and rider waiting costs are caused by the exact opposite phenomena: riders have to wait more when demand outstrips supply, but drivers have to wait more when supply outstrips demand. From now on, we will use "latency" to refer to rider latency: "lower latency" will automatically imply higher driver latency and vice versa. Rider demand is influenced by the delivered price. If p_i is the price on platform i and l_i the latency, the delivered price at platform i is $p_i + l_i$.

If $d_1 + d_2$ is the total demand served by the two platforms, the delivered price of platform i satisfies

$$p_i + l_i = 1 - d_1 - d_2.$$

While platform prices can differ, the delivered price on each platform is the same.

We model the driver supply curve as

$$s_1 + s_2 = w$$

In other words, if w is the average wage per driver, w is also then the total supply of drivers to both platforms. As platforms only pay drivers for rides taken, the total amount paid by platform i in wages is $w_i d_i$ and this must satisfy

$$w_i d_i = s_i(s_1 + s_2)$$

This expression is the platform's total labor costs. It is also the platform's total cost, because under we assume matching is costless to the platform.

The difference between single-homing and multi-homing arises from the congestion effects that multi-homing incurs. This manifests itself in the latency term which differs between the two regimes, a fact described in greater detail below.

4 Singlehoming vs Multihoming

Under single-homing, latency on platform i is $l_i = \frac{d_i}{s_i}$. The functional form of this latency term is microfounded in Appendix A. Our results hold for latency that is directly proportional to the rider to driver ratio. Market demand under single-homing is characterized by

$$p_i = 1 - d_i - d_j - \frac{d_i}{s_i}.$$

This means platform i's total cost is $d_i \cdot w_i = s_i(s_i + s_j)$. The profit π_i of platform i will be

$$\pi_i = d_i \left(1 - d_1 - d_2 - \frac{d_i}{s_i}\right) - s_i(s_1 + s_2).$$

Under multi-homing, each platform can recruit drivers, but then both platforms share all drivers. As before, platforms only pay drivers for rides taken on the platform, so a platform's costs are determined in the same way as under single-homing. However, multi-homing changes the latency of each platform: since all drivers are shared between platforms, latency on each platform now depends on the *total* supply of drivers, $s_1 + s_2$. Furthermore, multi-homing introduces cross-platform externalities between riders themselves: a rider on platform 2 causes congestion for a rider on platform 1, because they take the services of a driver who would otherwise be serving the rider of platform 1. Thus, latency for each platform depends on the demand of riders for both platforms, $d_1 + d_2$. Thus, the rider latency term under multi-homing is

$$l_1 = l_2 = \frac{d_1 + d_2}{s_1 + s_2}.$$

Under multi-homing, both platforms offer the same latency. This creates a market characterized by a single price, but varying wages for drivers:

$$p = 1 - d_1 - d_2 - \frac{d_1 + d_2}{s_1 + s_2},$$

$$w_i = \frac{s_i(s_i + s_j)}{d_i}.$$

Therefore, each firm's profit under multi-homing is

$$\pi_i = d_i \left(1 - d_1 - d_2 - \frac{d_1 + d_2}{s_1 + s_2}\right) - s_i(s_1 + s_2).$$

It turns out that in this model where firms are symmetric, the welfare comparison between multi-homing and single-homing is clear.

Proposition 1. *Both riders and drivers achieve higher equilibrium surplus under multi-homing than under single-homing.*

To understand the intuition for Proposition 1, recall that there are two components to both rider and driver surplus: the price/wage of a ride, and the latency of the ride. multi-homing decreases the price of the ride but increases rider latency (i.e. time waiting for a ride), because it increases the demand that drivers have to service—the demand of two platforms rather than the demand of just one platform. However, riders value the lower price more than they dislike the higher wait time: as a result, the net effect of these two opposing changes on rider welfare is positive. Likewise, multi-homing decreases the wage of the ride but also decreases driver latency (i.e. time waiting for a passenger). Since drivers value the lower latency more than they dislike the lower wage, the net effect is again an increase in driver welfare.

This result rests upon riders and drivers caring asymmetrically about latency. This asymmetry holds because for riders, time spent waiting is an inconvenience independent of the price they pay—however, for drivers, time spent waiting *decreases future wages by reducing the total number of rides they can offer.*[1] As a result, latency has a *multiplicative* effect on driver utility, but only an additive effect on rider utility. Thus, multi-homing enhances welfare for both sides of the market because it gives cost improvements to riders who value them the most, and latency improvements to drives who value them the most.

[1]Appendix A translates this intuition into a more rigorous microfounding of the latency term d/s.

5 Asymmetric Efficiency

Here we suppose one platform, say platform 1, is more efficient than the other, in the sense that it is more efficient at matching drivers to riders than its rival. We model this with platform 1 having an efficiency factor $\alpha \leq 1$ that reduces its latency. Platform 2's efficiency factor is normalized to 1, so we only need to consider α pertaining to platform 2.

Under single-homing, the latencies of the platforms are given by

$$l_1 = \frac{\alpha d_1}{s_1},\ l_2 = \frac{d_2}{s_2}.$$

Prices and wages are determined as follows:

$$1 - d_1 - d_2 = p_1 + \alpha \frac{d_1}{s_1} = p_2 + \frac{d_2}{s_2}$$

$$w_1 = \frac{\alpha s_1(s_1 + s_2)}{d_1},\ w_2 = \frac{s_2(s_1 + s_2)}{d_2}$$

The profit of each platform are as follows:

$$\pi_1 = d_1 \left(1 - d_1 - d_2 - \frac{\alpha d_1}{s_1}\right) - \alpha s_1(s_1 + s_2)$$
$$\pi_2 = d_2 \left(1 - d_1 - d_2 - \frac{d_2}{s_2}\right) - s_2(s_1 + s_2)$$

Under multi-homing with asymmetric efficiency, latency will depend upon all riders on all platforms. However, riders on platform 1 impose a lower externality on other riders and drivers, because platform 1's is more efficient at matching riders and drivers. Thus,

$$l_1 = l_2 = \frac{\alpha d_1 + d_2}{s_1 + s_2}.$$

Demand, prices and wages are determined as follows:

$$1 - d_1 - d_2 = p_1 + \frac{\alpha d_1 + d_2}{s_1 + s_2} = p_2 + \frac{\alpha d_1 + d_2}{s_1 + s_2}$$

$$w_1 = \frac{\alpha s_1(s_1+s_2)}{d_1}, w_2 = \frac{s_2(s_1+s_2)}{d_2}$$

Then profit of each platform is as follows:

$$\pi_1 = d_1\left(1 - d_1 - d_2 - \frac{\alpha d_1 + d_2}{s_1+s_2}\right) - \alpha s_1(s_1+s_2)$$
$$\pi_2 = d_2\left(1 - d_1 - d_2 - \frac{\alpha d_1 + d_2}{s_1+s_2}\right) - s_2(s_1+s_2)$$

In the next proposition we compare single-homing and multi-homing in this asymmetric model: in particular, we show an alarming effect of multi-homing on the relationship between efficiency and profit.

Proposition 2. *Under multi-homing, platform 1's profits decline as* α *decreases from 1. Under single-homing, platform 1's profits increase as* α *decreases from 1.*

Since a decrease in α means platform 1 is becoming more efficient, Proposition 2 shows that under multi-homing, a firm actually loses profit as it becomes more efficient than its competitor. Therefore, multi-homing disincentivizes a platform from unilaterally improving their matching technology because it reduces their profit relative to the baseline ($\alpha = 1$). This is because under multi-homing, a platform's efficiency gains have positive spillovers for its competitor: if Uber's matching technology becomes more efficient, drivers gain more time to drive for Lyft, which enables Lyft to seize demand from Uber. Since firms are in Cournot competition with each other, this increase in demand for Lyft directly reduces Uber's profits. This increase in demand for Lyft outweighs the increase in demand for Uber, resulting in a net loss of profit for Uber caused by its efficiency improvement.

We now illustrate Proposition 2 graphically. In the graphs below, we restrict α to lie in $[0.93, 1]$.[2] Figure 1 shows that under multi-homing, platform 1's profit decreases as it becomes more efficient.

[2] If $\alpha < 0.92$, platform 2's profit are negative—in other words, the interior solution assumption behind using the FOCs does not hold. This is the shutdown threshold demonstrated in Figure 7.

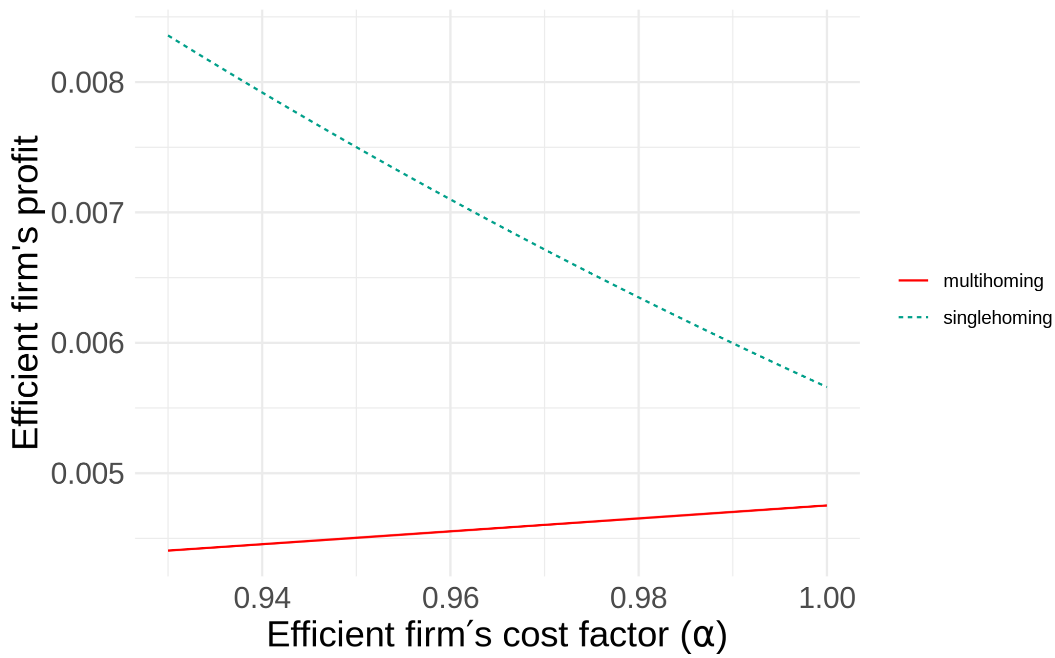

Figure 1: As α decreases, the efficient firm's profit increases under singlehoming but decreases under multi-homing.

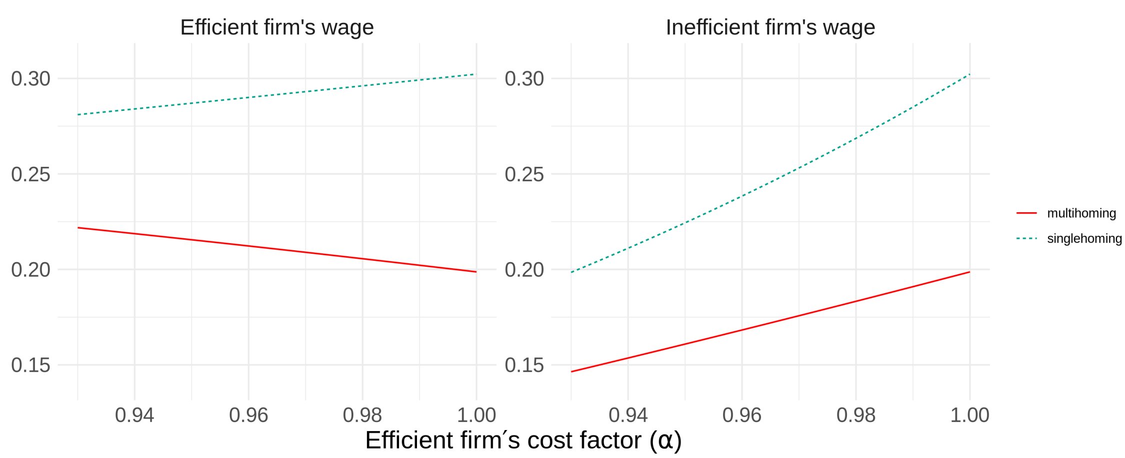

Figure 2: The wage that each firm offers as a function of α.

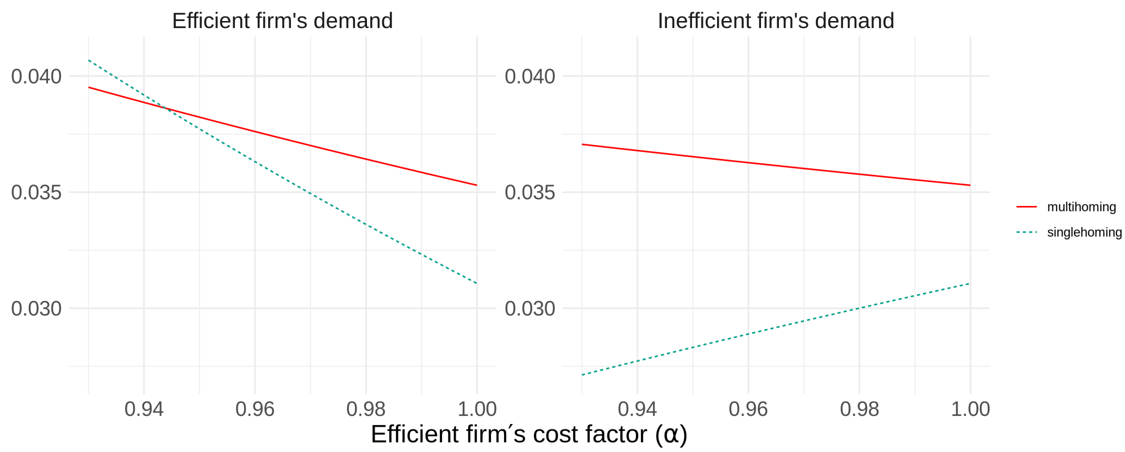

Figure 3: The demand that each firm services as a function of α.

In Figure 2 we see that this profit decrease is driven by an increase in costs: as platform 1 becomes more efficient, it actually pays more in wages! Recall that $w_i = \frac{\alpha s_i(s_1+s_2)}{d_i}$. Figure 3 shows that under single-homing, platform 1's demand increases sharply enough to outweigh increases in platform 1's supply (which would push wages up) and thus wages decrease. In contrast, under multi-homing, platform 1's demand rises more sluggishly, but their supply rises at the same rate as under single-homing: thus, wages are driven upwards and platform 1 faces higher costs than before.

To compound the effect on profitability, we see from Figure 4 that prices are significantly lower under multi-homing. Furthermore, since $p_i = 1-d_1-d_2-l_i$, we can see that when $\alpha = 1$, both firms have the same latency $l_1 = l_2 = \frac{d_1+d_2}{s_1+s_2}$ under multi-homing, both platforms must have the same price. Thus, platform 1 cannot efficiently extract surplus from riders through price changes: with any price increase, a large proportion of rider surplus will go to platform 2 instead, yielding smaller returns to price increases. This dual mechanism—increased wage cost and tied prices—results in lower profit for platform 1.

These figures also help us discount alternative explanations of these effects. Both platforms' optimal supply changes at the same rate with respect to α, so supply adjustments also cannot be driving the effect (Figure 5). Likewise, latency falls with lower α at approximately the same rate regardless of single-

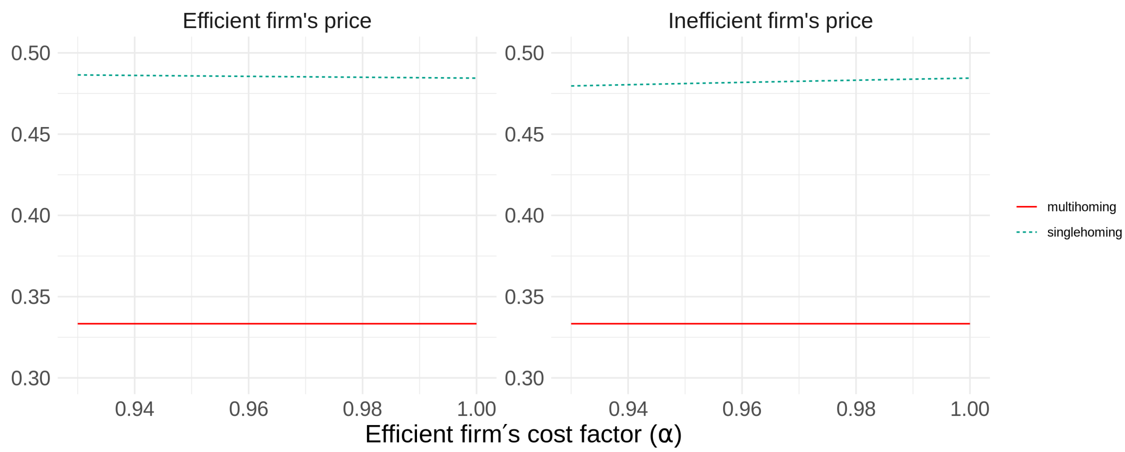

Figure 4: The price that each firm charges as a function of α.

Note: $p_i = 1 - d_1 - d_2 - l_i$.

homing or multi-homing, so it cannot be driving the profit reduction (Figure 6).

Prior work suggests that multi-homing reduces welfare in the short-term through the effect of lower prices or congestion (Bernstein, DeCroix, and Keskin 2018): Proposition 1 shows this is not the case when all market forces can adjust in equilibrium (as they can in our model). However, Proposition 2 suggests a novel channel through which multi-homing may hurt welfare in the long run: the decreased incentive for platforms to invest in their efficiency. In our model, as well as in most prior work, platforms are assumed to have symmetric and exogenous costs of matching. However, if that assumption is relaxed to allow for platforms to invest in their own efficiency, then multi-homing may discourage such investments.

5.1 Impact on Platform 2

Here we examine the impact on platform 2's profits as α decreases, i.e., platform 1 becomes more efficient. In each regime (single and multi-homing) if α becomes sufficiently small, the profits of platform 2 become negative, i.e., it exits. We make two assumptions:

1. There is no reentry or entry by other platforms—the monopoly persists.

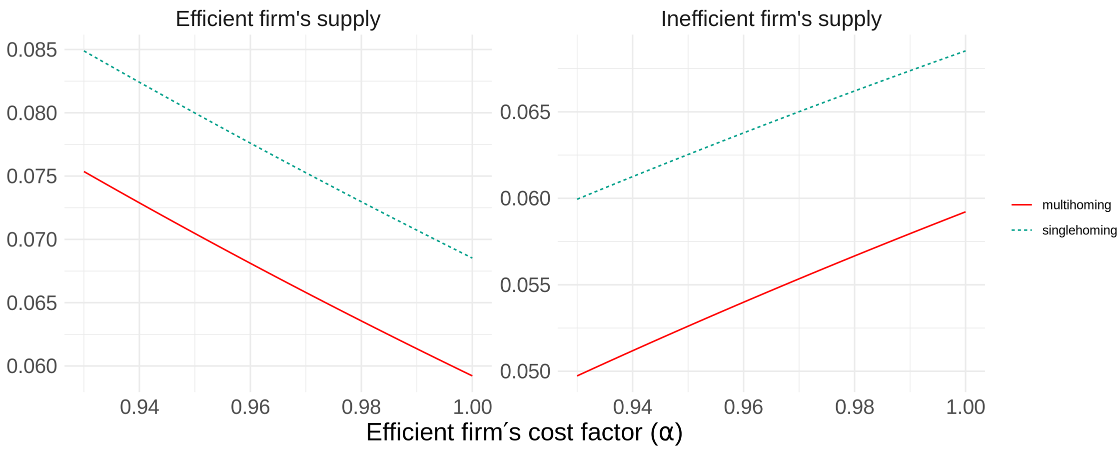

Figure 5: The supply that each firm hires as a function of α.

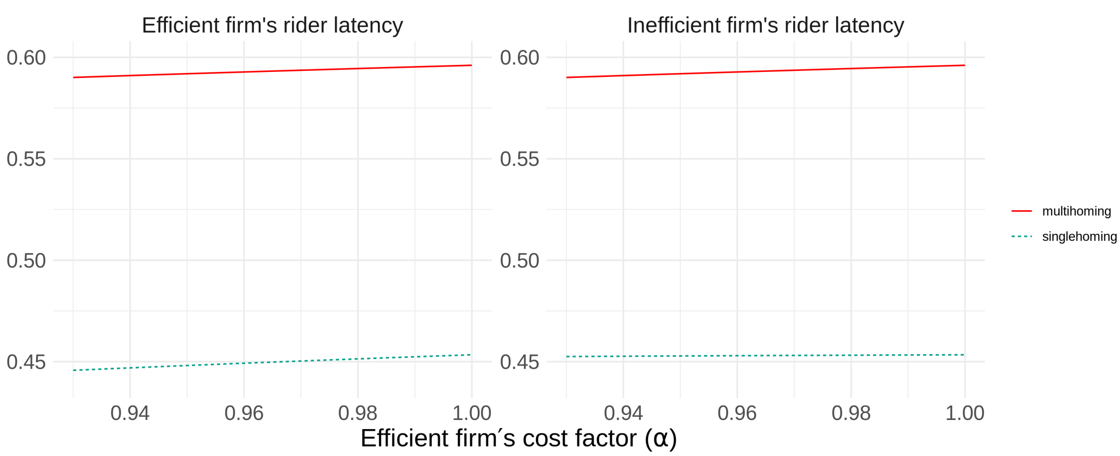

Figure 6: The latency that each firm's riders experience as a function of α.

Note: $l_1 = \alpha d_1/s_1$ under single-homing, $l_1 = (\alpha d_1 + d_2)/(s_1 + s_2)$ under multi-homing.

2. Platform 1 maintains its efficiency level α^* after achieving monopoly status.

Under monopoly, $d_2 = s_2 = 0$. As a consequence, we can drop subscripts and summarize demand, supply and profits at an efficiency level of α is as follows:

$$l = \frac{\alpha d}{s}$$

$$w = \frac{\alpha s^2}{d}$$

$$\implies \pi = d\left(1 - d - \frac{\alpha d}{s}\right) - \alpha s^2$$

Let α_t^* be the threshold α at which platform 2's profits become zero under market structure $t \in \{multi, single\}$. This is the "monopolization threshold"—if platform 1 attains this efficiency, firm 2 exits the market and the market becomes a monopoly.

Proposition 3. *The thresholds under multi-homing and single-homing satisfy*

$$\alpha^*_{multi} > \alpha^*_{single}$$

Furthermore, the monopoly with $\alpha = \alpha^*_{multi}$ reduces both driver and rider surplus relative to the monopoly with $\alpha = \alpha^*_{single}$

Figure 7 suggests—and Proposition 3 proves—that under multi-homing, platform 1 has a lower investment threshold needed to achieve monopoly status. Furthermore, a monopoly that emerges under multi-homing is socially inferior to one that emerges under single-homing. This result relates to an important phenomenon wherein ridesharing competition drives competitors out of the market. For example, both Uber and Didi Chuxing used to compete in the Chinese ridesharing market, but in August 2016, Uber exited the Chinese market and left Didi as the de facto monopolist—following which Didi promptly raised its prices. (SCMP 2016)

Together, these results capture the intuition that multi-homing redistributes surplus from the platforms to riders and drivers. The mechanism is also the

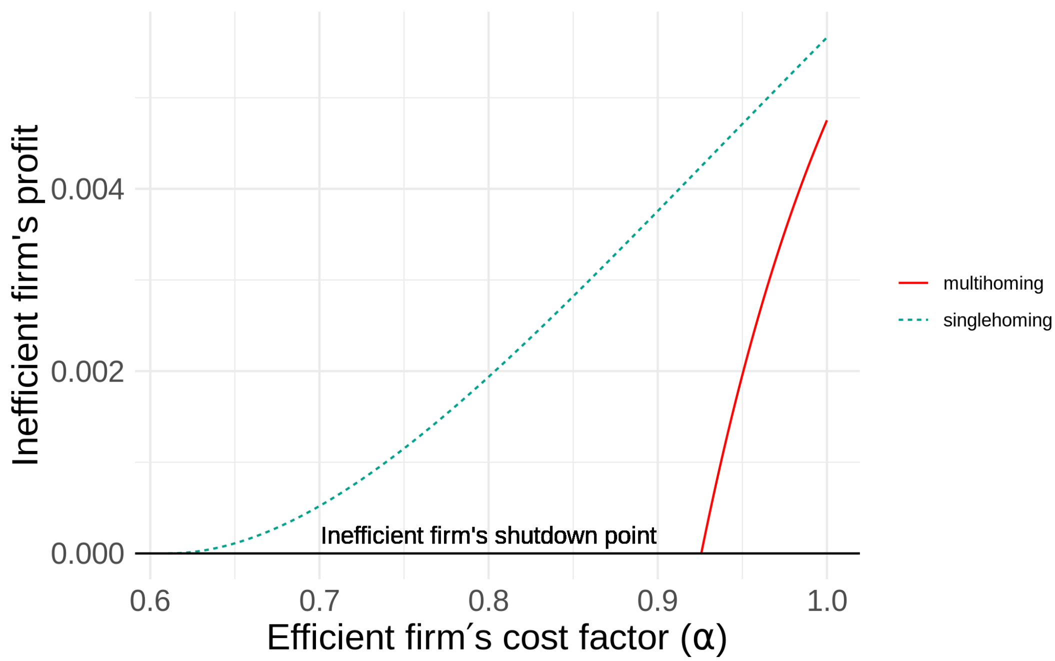

Figure 7: Illustrating $\alpha^*_{single} < \alpha^*_{multi}$: platform 2 shuts down at a much weaker efficiency threshold under multi-homing.

cause of Proposition 2: multi-homing intensifies the competition between platforms and increases spillover effects. Proposition 2 demonstrates the obvious upside of this redistribution: multi-homing is welfare-enhancing for both drivers and riders. Proposition 3, however, demonstrates the less-obvious danger of this redistribution: since platforms achieve lower profit under multi-homing, that also reduces the barrier to one platform driving out the other and achieving a harmful monopoly.

6 Conclusion

The ridesharing market is growing in importance, in its own right and as a model for other gig economy markets. As such, it presents novel and challenging questions around what market structure maximizes welfare. We have demonstrated three important facts: first, when prices and wages are allowed to adjust in equilibrium, multi-homing is welfare-enhancing for both riders and drivers. The second important fact is that increasing the efficiency of matching technology makes firms lose profit under multi-homing, but increases their profit under single-homing—a clear disincentive from investing in efficiency that exists only in multi-homing. The third important fact is that multi-homing reduces the threshold investment needed for one firm to monopolize the market.

Ridesharing platforms have been distinct in their focus on expansion and long-term profit. As a result, Proposition 2 and Proposition 3 suggest a new problem for the literature on ridesharing platforms to consider: how to balance rider/driver welfare with platform incentives to invest in the long-term. Future work could study this problem more precisely by expanding our model to formally include investment capacity, following a small body of work that expands matching market models to include investment (Hatfield, Kojima, and Kominers 2014). Our results have shown the effects of mechanical changes in α on profit and competition, which are beneficial because they do not rely on proving anything about endogenous investment. However, a model with endogenous investment would be valuable to go beyond what we can prove and show more clearly what tradeoffs exist between short-term rider/driver surplus and long-term investment incentives.

References

Angrist, Joshua D, Sydnee Caldwell, and Jonathan V Hall. 2017. *Uber vs. taxi: A drivers eye view.* Tech. rep. National Bureau of Economic Research.

Armstrong, Mark. 2006. "Competition in two-sided markets". *The RAND Journal of Economics* 37 (3): 668–691.

Armstrong, Mark, and Julian Wright. 2007. "Two-sided markets, competitive bottlenecks and exclusive contracts". *Economic Theory* 32 (2): 353–380.

Banerjee, Siddhartha, Ramesh Johari, and Carlos Riquelme. 2016. "Dynamic pricing in ridesharing platforms". *ACM SIGecom Exchanges* 15 (1): 65–70.

Bernstein, Fernando, Gregory DeCroix, and N Bora Keskin. 2018. "Competition between two-sided platforms under demand and supply congestion effects". *Available at SSRN 3250224.*

Bryan, Kevin A, and Joshua S Gans. 2019. "A theory of multihoming in rideshare competition". *Journal of Economics & Management Strategy* 28 (1): 89–96.

Castillo, Juan Camilo, Dan Knoepfle, and Glen Weyl. 2017. "Surge pricing solves the wild goose chase". In *Proceedings of the 2017 ACM Conference on Economics and Computation*, 241–242. ACM.

Chen, M Keith, and Michael Sheldon. 2016. "Dynamic Pricing in a Labor Market: Surge Pricing and Flexible Work on the Uber Platform." In *Ec*, 455.

Choi, Jay Pil. 2010. "Tying in two-sided markets with multi-homing". *The Journal of Industrial Economics* 58 (3): 607–626.

Cohen, Peter, Robert Hahn, Jonathan Hall, Steven Levitt, and Robert Metcalfe. 2016. *Using big data to estimate consumer surplus: The case of uber.* Tech. rep. National Bureau of Economic Research.

Cramer, Judd, and Alan B Krueger. 2016. “Disruptive change in the taxi business: The case of Uber”. *American Economic Review* 106 (5): 177–82.

Evans, David S, and Richard Schmalensee. 2005. *The industrial organization of markets with two-sided platforms*. Tech. rep. National Bureau of Economic Research.

Gabszewicz, Jean J, and Xavier Wauthy. 2004. “Two-sided markets and price competition with multi-homing”. *Available at SSRN 975897*.

Hall, Jonathan V, and Alan B Krueger. 2018. “An analysis of the labor market for Uberfffdfffdfffds driver-partners in the United States”. *ILR Review* 71 (3): 705–732.

Hatfield, John William, Fuhito Kojima, and Scott Duke Kominers. 2014. “Investment incentives in labor market matching”. *American Economic Review* 104 (5): 436–41.

Liu, Qihong, Oksana Loginova, and X Henry Wang. 2017. “The Impact of Multi-Homing in a Ride-Hailing Market”. *Available at SSRN 2968504*.

Nikzad, Afshin. 2017. “Thickness and competition in ride-sharing markets”. *Available at SSRN 3065672*.

Rochet, Jean-Charles, and Jean Tirole. 2003. “Platform competition in two-sided markets”. *Journal of the european economic association* 1 (4): 990–1029.

SCMP. 2016. “Ride hailing giant Didi Chuxing raises prices in Chinafffdfffdfffds biggest cities”. Visited on 09/12/2020. https://www.scmp.com/business/china-business/article/2015137/ride-hailing-giant-didi-chuxing-raises-prices-chinas-biggest.

Statista. 2019. “Ridesharing Services in the US: Statistics and Facts”. Visited on 07/30/2020. https://www.statista.com/topics/4610/ridesharing-services-in-the-us/.

Uber. 2014. “A Deeper Look at Uber’s Dynamic Pricing Model”. Visited on 08/20/2020. http://abovethecrowd.com/2014/03/11/a-deeper-look-at-ubers-dynamic-pricing-model/.

A Microfounding the Latency Model

We describe a model to suggest why latency should scale with $\frac{d}{s}$. Suppose there are d passengers and s drivers. Imagine that each passenger is assigned with equal probability to one of the drivers independently of the others. Then, the number of passengers a driver gets is Binomial with parameters $(d, \frac{1}{s})$. Thus, the expected number of passengers a driver gets is $\frac{d}{s}$. If w is the wage per ride that the driver receives, her expected wage is $\frac{wd}{s}$. The assumption we are making is that each driver gets a bunch of rides and services them consecutively without delay. When done, that is the end of business for them. Thus, drivers never suffer a delay cost. This is also equivalent to assuming that passenger demand is shared equally between drivers.

Suppose each driver has an opportunity cost that is uniform on $[0, 1]$. Then, at wage w only drivers with an opportunity cost $\leq \frac{wd}{s}$ will be available. Hence, the expected number of available drivers, s, should equal $\frac{wd}{s}$.

On the passenger side, consider a random passenger with a cost c per unit of time to wait for a ride. If p is the price of a ride and they had to wait t time units, the delivered price will be $p + ct$. Assume passengers divided equally between drivers. Then, each driver is carrying $\frac{d}{s}$ passengers. Assume each carried passenger is equally likely to to be the first, second third passenger in the queue. Assume also, that each passenger takes unit time to be dropped off. Then, a passengers expected delay will be $\sum_{j=1}^{d/s} \frac{j}{d/s} = \frac{d/s+1}{2}$.

If we normalize every passengers waiting cost to 1, i.e. $c = 1$, then assuming a uniform distribution over reservation prices:

$$d = 1 - (p + t) \;\Rightarrow p = 1 - d - \frac{d/s + 1}{2} = \frac{1}{2} - d - \frac{d}{2s}.$$

This latency term of $d/2s$ can be normalized to the d/s used in the main body—the goal is simply to demonstrate the reason for a latency term which is directly proportional to d and inversely proportional to s.

B Proof of Proposition 1

To prove Proposition 1, we will separately analyze the equilibrium of the single-homing market and the multi-homing market, then compare rider surplus and driver surplus achieved under each, showing that both are higher under multi-homing. These calculations can be verified using our online replication archive.

B.1 Singlehoming Equilibrium

Recall that under single-homing, each firm's profit is

$$\pi_i = d_i \left(1 - d_1 - d_2 - \frac{d_i}{s_i}\right) - s_i(s_1 + s_2)$$

Furthermore, we can compute the rider surplus for any equilibrium d, s:

$$\begin{aligned}
RS &= \int_0^{2d} 1 - 2x - \frac{x}{2s} - p \, dx \\
&= \int_0^{2d} 1 - 2x - \frac{x}{2s} - 1 + 2x + \frac{x}{s} \, dx \\
&= \int_0^{2d} \frac{x}{2s} dx \\
&= \frac{d^2}{s}
\end{aligned}$$

Likewise, the driver surplus is

$$\begin{aligned}
DS &= \int_0^{2s} w - \frac{x^2}{d} \, dx \\
&= \int_0^{2s} \frac{2x^2}{d} - \frac{x^2}{d} \, dx \\
&= \int_0^{2s} \frac{x^2}{d} \, dx \\
&= \frac{8s^3}{3d}
\end{aligned}$$

The FOCs that yield an interior solution in the market are

$$
\begin{aligned}
0 &= \frac{\partial \pi_1}{\partial d_1} = 1 - 2d_1 - d_2 - \frac{2d_1}{s_1} - d_2 \\
0 &= \frac{\partial \pi_1}{\partial s_1} = \frac{d_1^2}{s_1^2} - 2s_1 - s_2 \\
0 &= \frac{\partial \pi_2}{\partial d_2} = 1 - 2d_2 - d_1 - \frac{2d_2}{s_2} - d_1 \\
0 &= \frac{\partial \pi_2}{\partial s_2} = \frac{d_2^2}{s_2^2} - 2s_2 - s_1
\end{aligned}
$$

The only solution to this system of equations (verifiable in any solver) is

$$d_1 = d_2 = 0.0312,\ s_1 = s_2 = 0.0685$$

Then the profit of each firm is

$$\pi_1 = \pi_2 = 0.0312 \left(1 - 2 \times 0.0312 - \frac{0.0312}{0.0685}\right) - 0.0685^2 \times 2 = 0.0057$$

Then rider surplus is

$$
\begin{aligned}
RS &= \int_0^{0.0624} [1 - x - \frac{d^*}{s^*} - p^*] dx \\
&= [x - \frac{x^2}{2} - 0.45x - 0.482x]_0^{0.0624} \\
&= 0.0023
\end{aligned}
$$

Similarly, driver surplus is

$$
\begin{aligned}
DS &= 2s^* w^* - \int_0^{2s^*} \frac{x^2}{d^*} dx \\
&= 2 \cdot 0.0685 \cdot 0.3 - [\frac{x^3}{3d^*}]_0^{0.137} \\
&= 0.0411 - \frac{0.137^3}{0.0936} \\
&= 0.0136
\end{aligned}
$$

B.2 Multihoming Equilibrium

Recall that under multi-homing, each firm's profit is

$$\pi_i = d_i \left(1 - d_1 - d_2 - \frac{d_1 + d_2}{s_1 + s_2}\right) - s_i(s_1 + s_2)$$

The FOCs are

$$\begin{aligned}
0 &= \frac{\partial \pi_1}{\partial d_1} = 1 - 2d_1 - d_2 - \frac{2d_1 + d_2}{s_1 + s_2} \\
0 &= \frac{\partial \pi_1}{\partial s_1} = \frac{d_1(d_1 + d_2)}{(s_1 + s_2)^2} - 2s_1 - s_2 \\
0 &= \frac{\partial \pi_2}{\partial d_2} = 1 - 2d_2 - d_1 - \frac{2d_2 + d_1}{s_1 + s_2} \\
0 &= \frac{\partial \pi_2}{\partial s_2} = \frac{d_2(d_1 + d_2)}{(s_1 + s_2)^2} - 2s_2 - s_1
\end{aligned}$$

The only real solution to this system (verifiable in any solver) is

$$d_1 = d_2 = d = 0.035, s_1 = s_2 = s = 0.059$$

Then we have price given by

$$p^* = 1 - 2d - \frac{d}{s} = 1 - 0.07 - \frac{0.035}{0.059} = 0.93 - 0.6 = 0.33$$

The market wage is then given by

$$w^* = \frac{2s^2}{d} = \frac{0.059^2}{0.035} = 0.199$$

Profit is thus symmetric and

$$\pi_1^* = \pi_2^* = 0.035(0.33 - 0.199) = 0.004$$

Variable	Singlehoming	Multihoming
Rider demand	0.0312	0.035
Driver supply	0.0685	0.059
Rider prices	0.482	0.33
Driver wages	0.3	0.199
Rider latency	0.455	0.593
Driver latency	2.195	1.685
Rider surplus	**0.0023**	**0.026**
Driver surplus	**0.0136**	**0.015**
Firm profit	0.0057	0.004

Table 1: The solution for key variables under each equilibrium, and how the change under multi-homing affects rider and driver surplus.

Rider surplus is

$$\begin{aligned}
RS &= \int_0^{0.07} \left(1 - x - \frac{0.035}{0.059} - 0.33\right) dx \\
&= \int_0^{0.07} (0.407 - x) dx = (0.407 * 0.07 - 0.07^2/2) \\
&= 0.026
\end{aligned}$$

Driver surplus is

$$\begin{aligned}
DS &= 2 * 0.059 * 0.199 / - \int_0^{0.118} \frac{x^2}{0.07} dx \\
&= 0.023 - \frac{0.118^3}{3 * 0.07} = 0.023 - 0.008 \\
&= 0.015
\end{aligned}$$

We can summarize the difference between the single-homing equilibrium and the multi-homing equilibrium in Table 1. We can see that under multi-homing, both rider surplus and driver surplus are higher in equilibrium, which proves Proposition 1.

C Proof of Proposition 2

Unlike with Proposition 1, we cannot solve for equilibrium directly: there are four equations but five unknowns $(d_1, d_2, s_1, s_2, \alpha)$. Instead we aim to study $\frac{\partial \pi_1}{\partial \alpha}$: if $\frac{\partial \pi_1}{\partial \alpha} > 0$, that means a lower α (higher efficiency) leads to lower profit. Likewise, if $\frac{\partial \pi_1}{\partial \alpha} < 0$ that means lower α leads to higher profit. Thus, Proposition 2 can be reframed as saying

$$\frac{\partial \pi_1}{\partial \alpha} < 0 \text{ under single-homing}$$

$$\frac{\partial \pi_1}{\partial \alpha} > 0 \text{ under multi-homing}$$

We can prove each of these. The general profit expression in both markets is

$$\pi_1 = d_1(1 - d_1 - d_2 - l_1) - \alpha s_1(s_1 + s_2)$$

Then we can study how the FOCs vary in each market structure.

C.1 Singlehoming

Under single-homing, recall that each firm's profit is given by

$$\pi_1 = d_1 \left(1 - d_1 - d_2 - \frac{\alpha d_1}{s_1}\right) - \alpha s_1(s_1 + s_2)$$
$$\pi_2 = d_2 \left(1 - d_1 - d_2 - \frac{d_2}{s_2}\right) - s_2(s_1 + s_2)$$

Then the relevant FOCs are

$$\frac{\partial \pi_1}{\partial d_1} = 1 - 2d_1 - d_2 - 2\alpha \frac{d_1}{s_1} = 0$$
$$\frac{\partial \pi_1}{\partial s_1} = \alpha \frac{d_1^2}{s_1^2} - \alpha(2s_1 + s_2) = 0$$
$$\frac{\partial \pi_2}{\partial d_2} = 1 - d_1 - 2d_2 - 2\frac{d_2}{s_2} = 0$$
$$\frac{\partial \pi_2}{\partial s_2} = \frac{d_2^2}{s_2^2} - (s_1 + 2s_2) = 0$$

We can characterize $\partial \pi_1 / \partial \alpha$ by first characterizing the relationship $d_1'(\alpha)$, $d_2'(\alpha)$, $s_1'(\alpha)$, $s_2'(\alpha)$. We will suppress notation and refer to these as d_1', d_2', s_1', s_2'.

Lemma 1. *Under single-homing, $d_1', s_1' < 0$ while $d_2', s_2' > 0$.*

Proof. Consider only firm 1's maximization problem, holding d_2, s_2 fixed. Then from the second FOC, we can cancel α to get

$$\frac{d_1^2}{s_1^2} - 2s_1 - s_2 = 0$$

$$\implies d_1 = s_1\sqrt{2s_1 + s_2}$$

Note that we can assume roots are positive henceforth because $d_1, s_1 > 0$. We can substitute this into the first FOC to get

$$1 - 2s_1\sqrt{2s_1 + s_2} - d_2 - 2\alpha\sqrt{2s_1 + s_2} = 0$$

$$\implies 2(s_1 + \alpha)\sqrt{2s_1 + s_2} = 1 - d_2$$

Implicitly differentiate this with respect to α to see

$$2s_1'\sqrt{2s_1 + s_2} + 2\sqrt{2s_1 + s_2} + s_1' \frac{2(s_1 + \alpha)}{\sqrt{2s_1 + s_2}} = 0$$

$$\implies 2(s_1' + 1)(2s_1 + s_2) + 2s_1'(s_1 + \alpha) = 0$$

$$\implies s_1'(3s_1 + s_2 + \alpha) = -2s_1 - s_2$$

$$\implies s_1' = \frac{-2s_1 - s_2}{3s_1 + s_2 + \alpha}$$

The numerator is strictly negative, the denominator is strictly positive and so $s_1' < 0$.

After that, implicitly differentiate the first FOC on its own.

$$1 - 2d_1 - d_2 - \frac{2\alpha d_1}{s_1} = 0$$

$$\implies 2d_1(1 + \frac{\alpha}{s_1} = 1 - d_2$$

$$\implies d_1 = \frac{s_1(1 - d_2)}{2(s_1 + \alpha)}$$

$$\implies d_1' = \frac{2(s_1 + \alpha)s_1'(1 - d_2) - 2s_1(1 - d_2)(1 + s_1')}{4(s_1 + \alpha)^2}$$

$$= (1 - d_2)\left(\frac{s_1'}{2(s_1 + 1)} - \frac{2s_1 + 2s_1 s_1'}{4(s_1 + 1)^2}\right)$$

$$= (1 - d_2)\left(\frac{2s_1 s_1' + 2s_1' - 2s_1 s_1' - 2s_1}{4(s_1 + 1)^2}\right)$$

$$\implies d_1' = \frac{(s_1' - s_1)(1 - d_2)}{2(s_1 + 1)^2}$$

Note that $d_2 < 1, s_1 + 1 > 0$ so every term is positive except $s_1' - s_1$, which is negative ($s_1' < 0$ as shown previously, and $s_1 > 0$). Thus, the overall fraction is negative and so

$$s_1' < 0, d_1' < 0$$

We can perform a symmetric analysis for firm 2, but this time allowing s_1, d_1 to vary. From the fourth FOC,

$$d_2 = s_2\sqrt{2s_2 + s_1}$$

Substituting into the third FOC yields

$$1 - d_1 - 2s_2\sqrt{2s_2 + s_1} - 2\sqrt{2s_2 + s_1} = 0$$

$$\implies 2(s_2+1)\sqrt{2s_2+s_1} = 1-d_1$$

Then implicitly differentiate both with respect to α to see

$$2s_2'\sqrt{2s_2+s_1} + 2(s_2+1)\frac{2s_2'+s_1'}{2\sqrt{2s_2+s_1}} = -d_1'$$

$$\implies 2s_2'(2s_2+s_1) + (2s_2+2)(2s_2'+s_1') = -d_1\sqrt{2s_2+s_1}$$

$$\implies s_2'(8s_2+2s_1+4) = -s_1'(2s_2+2) - d_1'\sqrt{2s_2+s_1}$$

$$\implies s_2' = \frac{-s_1'(2s_2+2) - d_1'\sqrt{2s_2+s_1}}{8s_2+2s_1+4}$$

Since we just showed $s_1', d_1' < 0$ the numerator and denominator are both positive, so $s_2' > 0$.

After that, implicitly differentiate the third FOC:

$$1 - d_1 - 2d_2 - \frac{2d_2}{s_2} = 0$$

$$\implies -d_1' - 2d_2' - \frac{2d_2'}{s_2} + \frac{2d_2 s_2'}{s_2^2} = 0$$

$$\implies d_2'(2+\frac{2}{s_2}) = -d_1' + \frac{2d_2 s_2'}{s_2^2}$$

$$\implies d_2' = \frac{-d_1' + \frac{2d_2 s_2'}{s_2^2}}{2+\frac{2}{s_2}}$$

Since $s_2' > 0, d_1' < 0$ the numerator and denominator are both positive so $d_2' > 0$. $\square$

Lemma 1 allows us to analyze the profit derivative directly.

$$\frac{\partial \pi_1}{\partial \alpha} = d_1' - 2d_1d_1' - d_2d_1' - d_1d_2' - \frac{d_1^2}{s_1} - \left(\frac{\alpha d_1'}{s_1} - \frac{\alpha s_1' d_1^2}{s_1^2}\right) - s_1(s_1+s_2) - \alpha(2s_1s_1' + s_1s_2' + s_1's_2)$$

$$= d_1'\left(1 - 2d_1 - d_2 - \frac{2\alpha d_1}{s_1}\right) - d_1d_2' - \frac{d_1^2}{s_1} + s_1'\left(\frac{\alpha d_1^2}{s_1^2} - \alpha(2s_1+s_2)\right) - s_1(s_1+s_2) - \alpha s_1 s_2'$$

The bracketed terms are zero (by the first two FOCs), which allows us to simplify this expression dramatically, to become

$$\frac{\partial \pi_1}{\partial \alpha} = -d_1 d_2' - \frac{d_1^2}{s_1} - s_1(s_1 + s_2) - \alpha s_1 s_2'$$

We know from Lemma 1 that $d_2' > 0, s_2' > 0$ and so every term is negative, so

$$\frac{\partial \pi_1}{\partial \alpha} < 0$$

Thus, we've shown that as firm 1 become more efficient (α decreases), π_1 increases so firm 1 becomes more profitable. Thus, firms are incentivized to invest in their matching efficiency under single-homing.

C.2 Multihoming

The procedure from before works in the multi-homing market as well. Each platform's profit will be

$$\pi_1 = d_1 \left(1 - d_1 - d_2 - \frac{\alpha d_1 + d_2}{s_1 + s_2} - \frac{\alpha s_1 (s_1 + s_2)}{d_1} \right)$$

$$\pi_2 = d_2 \left(1 - d_1 - d_2 - \frac{\alpha d_1 + d_2}{s_1 + s_2} - \frac{s_2 (s_1 + s_2)}{d_2} \right)$$

Then the relevant FOCs are

$$\begin{aligned}
\frac{\partial \pi_1}{\partial d_1} &= 1 - 2d_1 - d_2 - \frac{2\alpha d_1 + d_2}{s_1 + s_2} \\
\frac{\partial \pi_1}{\partial s_1} &= \frac{d_1(\alpha d_1 + d_2)}{(s_1 + s_2)^2} - \alpha(2s_1 + s_2) \\
\frac{\partial \pi_2}{\partial d_2} &= 1 - d_1 - 2d_2 - \frac{\alpha d_1 + 2d_2}{s_1 + s_2} \\
\frac{\partial \pi_2}{\partial s_2} &= \frac{d_2(\alpha d_1 + d_2)}{(s_1 + s_2)^2} - (s_1 + 2s_2)
\end{aligned}$$

The key difference in our analysis from the single-homing market is that previously, we proved $\frac{\partial \pi_1}{\partial \alpha} < 0$ independent of the value of α. However, the framing of Proposition 2 is about becoming more efficient than the other firm: thus, we only need to evaluate the derivative at $\alpha = 1$ to capture this effect. This will greatly simplify the analysis, in part because $\alpha = 1$ is just the special case where firms are symmetric, and the equilibrium that we solved for in Proposition 1 holds there—the specific values can be referenced from Table 1, and we know

$$s_1 = s_2 = 0.059, d_1 = d_2 = 0.035$$

Then we can prove a similar lemma to before.

Lemma 2. *Under multi-homing,*

$$d_1'(1) = -0.0316, d_2'(1) = -0.0078, s_1'(1) = -0.0197, s_2'(1) = 0.2291$$

Proof. From the second FOC,

$$\alpha d_1^2 + d_2 d_1 + (-\alpha s_1^2 - \alpha s_1(s_1 + s_2)^3) = 0$$

By the quadratic formula,

$$d_1 = \frac{-d_2 \pm \sqrt{d_2^2 + 4\alpha^2(s_1^2 + s_1(s_1 + s_2)^3}}{2\alpha}$$

Note that we only need to consider the positive solution, since $d_1 > 0$. Take the derivative and evaluate at $\alpha = 1$:

$$d_1' =$$

Since $2\alpha d_1$ is the numerator above and also present in the first FOC, substituting this strategically into the first FOC yields

$$1 - 2d_1 - d_2 - \frac{\sqrt{d_2^2 + 4\alpha^2(s_1^2 + s_1(s_1 + s_2)^3}}{s_1 + s_2} = 0$$

Implicitly differentiate this with respect to α to get

$$-2d_1' - \frac{(s_1+s_2)(2d_1+2\alpha d_1') - s_1'(2\alpha d_1 + d_2)}{(s_1+s_2)^2} = 0$$

$$\implies s_1' = \frac{(s_1+s_2)(2d_1+2\alpha d_1')}{2\alpha d_1 + d_2} - \frac{(2d_1'+d_2)(s_1+s_2)^2}{2\alpha d_1 + d_2}$$

□

Note that this is analogous in sign to Lemma 1 except for d_2': whereas $d_2' > 0$ under single-homing, $d_2' < 0$ under multi-homing. In other words, multi-homing causes firm 2's demand to increase in response to efficiency by firm 1.

$$\pi_1 = d_1 - d_1^2 - d_1 d_2 - \frac{\alpha d_1^2 + d_1 d_2}{s_1+s_2} - \alpha s_1^2 - \alpha s_1 s_2$$

$$\implies \frac{\partial \pi_1}{\partial \alpha} = d_1' - 2d_1 d_1' - d_1' d_2 - d_1 d_2' - \frac{(s_1+s_2)(2\alpha d_1 d_1' + d_1^2 + d_1' d_2 + d_1 d_2') - (\alpha d_1^2 + d_1 d_2)(s_1' + s_2')}{(s_1+s_2)^2}$$
$$-s_1^2 - 2\alpha s_1 s_1' - s_1 s_2 - \alpha s_1' s_2 - \alpha s_1 s_2'$$

Note that grouping together terms with d_1', s_1' yields

$$d_1'\left(1 - 2d_1 - d_2 - \frac{2\alpha d_1 + d_2}{s_1+s_2}\right), s_1'\left(\frac{a d_1^2 + d_1 d_2}{(s_1+s_2)^2} - \alpha(2s_1+s_2)\right)$$

By the first two FOCs, both of the bracketed expressions are zero and can be cancelled out. Thus,

$$\frac{\partial \pi_1}{\partial \alpha} = -d_1 d_2' - \frac{d_1(d_1+d_2')}{s_1+s_2} + s_2' \frac{d_1(\alpha d_1 + d_2)}{(s_1+s_2)^2} - s_1^2 - s_1 s_2 - \alpha s_1 s_2'$$

Plugging in values of the symmetric equilibrium from Table 1, as well as the derivatives from Lemma 2 gives us that

$$\left.\frac{\partial \pi_1}{\partial \alpha}\right|_{\alpha=1} = -d \cdot d_2' - \frac{d(d+d_2')}{2s} + \frac{s_2' \cdot d^2}{2s^2} - 2s^2 - s \cdot s_2'$$

$$= -0.035 \cdot d_2' - \frac{0.035(0.035 + d_2')}{0.118} + \frac{0.035^2 \cdot s_2'}{2 \cdot 0.059^2} - 2 \cdot 0.059^2 - 0.059 \cdot s_2'$$

$$\implies \frac{\partial \pi_1}{\partial \alpha} = 0.0122$$

Thus, $\partial \pi_1 / \partial \alpha |_{\alpha=1} > 0$, so when α decreases from 1, π_1 also decreases. This completes the proof of Proposition 2.

D Proof of Proposition 3

First, we must find the threshold α^* at which firm 2 shuts down. This is the α^* satisfying the condition

$$d_2(\alpha^*) \cdot (1 - d_1(\alpha^*) - d_2(\alpha^*) - l(\alpha^*)) - d_2(\alpha^*) \cdot w_2(\alpha^*) = 0$$

We cannot solve this analytically, because there are five unknowns (d_1, d_2, s_1, s_2, α^*) and only four FOCs. However, we can solve this numerically by tracing firm 2's profit π_2 as α changes, and identifying the zero-profit point with arbitrary numerical precision. This is shown in Figure 7. Immediately, we can see that multi-homing has a significantly weaker threshold α^* to induce monopoly compared to single-homing. In other words, multi-homing competition is "less stable" than single-homing competition, in the sense that firms under multi-homing are much more at risk of being bankrupted by their competitor's efficiency gains. This aligns with Table 1, which shows that at the baseline (symmetry), each firm makes higher profit under single-homing than under multi-homing. These solutions, and all calculations below, can all be verified in our replication archive.

The numerical solution proves the first part of Proposition 3, which is that $\alpha^*_{single} < \alpha^*_{multi}$. From the numerical solutions, we can see the thresholds α^* are

$$\alpha^*_{multi} = 0.9257, \alpha^*_{single} = 0.6104$$

Given a value of α^*, the monopoly solution can be found through the FOCs:

$$\frac{\partial \pi}{\partial d} = 1 - 2d - \frac{2\alpha^* d}{s} = 0$$

$$\frac{\partial \pi}{\partial s} = \frac{\alpha^* d^2}{s^2} - 2\alpha^* s = 0$$

Then the solution to this pair of equations, for each value of α^*, can be found as

$$\alpha^* = 0.9257 \implies d = 0.055, s = 0.115$$

$$\alpha^* = 0.6104 \implies d = 0.120, s = 0.193$$

We can use these parameters to compute rider and driver surplus under each asymmetry-induced monopoly.

$$\alpha^* = 0.9257 \implies RS = 0.0138, DS = 0.0171$$

$$\alpha^* = 0.6104 \implies RS = 0.0301, DS = 0.0245$$

This calculation proves the second part of Proposition 3, on the welfare comparison between the two monopolies.